DISTURBANCES IN AN ELECTROMAGNETIC FIELD CAUSED BY SHOCK WAVES UNDER THE INFLUENCE OF A SUDDEN INCREASE IN CONDUCTIVITY

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Strong shock-wave propagation through a gas involves very sharp changes in the properties of the gas after the shock front has passed; pressure, temperature, density, electrical conductivity and other parameters of the medium undergo a very marked increase.

If the shock wave is propagated within a space where there are magnetic and electrostatic fields, the change in properties of the medium due to the passage of the shock-wave front causes disturbances in the magnetic and electrical fields which, themselves, will be propagated as electromagnetic waves. Problems associated with the radiation of electromagnetic waves caused by shock waves have been dealt with by several authors [1-4].

In this paper we discuss the problem of electromagnetic wave radiation resulting from the propagation of spherical shock waves in weak electric and magnetic fields. It will be assumed that the excitation of the electromagnetic waves is associated with the jump in conductivity due to the passage of the shock wave through the gas. The case of plane wave propagation has already been dealt with in [2,3].

Assume that a strong shock wave propagates at a velocity $D(t)$, where t is the time, the gas occupying an infinitely large volume. As the electric and magnetic fields ahead of the shock-wave front are assumed to be weak, their effect will be neglected behind the shock-wave front.

This means that the parameters of the wave front will be the same as those obtained from a gasdynamic solution to the problem. It is assumed that the gasdynamic problem has been completely solved and the velocity $D(t)$ is therefore known.

Let us deal with the initial and boundary conditions.

Suppose at an initial instant $t = 0$ the vectors of magnetic field intensity $\mathbf{H}$ and electric field intensity $\mathbf{E}$ are constant. We will assume that the conductivity of the gas ahead of the front is zero, whilst to the rear it is infinite. Because there are no magnetic charges at the shock-wave front, we can apply the condition of continuity to the normal component of magnetic field strength $\mathbf{H}_n$ (magnetic permeability assumed to be unity). It follows from Maxwell's equation that the tangential component of the electric field $\mathbf{E}_\tau$ is also continuous. It should be observed that the continuity conditions on $\mathbf{H}_n$ and $\mathbf{E}_\tau$ correspond to similar conditions on shock waves propagated through a medium of infinite conductivity [4].

By analogy with the case in which the electric and magnetic fields are parallel to the wave front, as discussed in [2], we consider the tangential component of the magnetic field $\mathbf{H}_\tau$ to be continuous. It is considered, also, as in [2], that within the shock layer itself the coefficient of magnetic viscosity ν_m exceeds other dissipative coefficients. We use suffix 1 to denote quantities ahead of the shock front, 2 immediately behind, and we can then write

$$\mathbf{H}_1 = \mathbf{H}_2, \qquad \mathbf{E}_{1\tau} = \mathbf{E}_{2\tau} \tag{1}$$

Because of infinite conductivity of the gas behind the shock front, in a coordinate system based on the wave we have the following relation between $\mathbf{H}$ and $\mathbf{E}$:

$$\mathbf{E}_2 = -\frac{1}{c}\,\mathbf{v}_2 \times \mathbf{H}_2 \qquad (c \text{ is the velocity of light})$$

Taking account of (1), in a stationary coordinate system, we have

$$\mathbf{E}_{1\tau} = -\frac{1}{c}\,[(\mathbf{v}_2 - \mathbf{D}) \times \mathbf{H}_1]_\tau \tag{2}$$

The boundary conditions (1) and (2) at the shock front should be satisfied by the solutions of the electromagnetic wave propagation problem. As the solution of the gasdynamic problem is assumed to be known, the quantities $\mathbf{v}_2$ and $\mathbf{D}$ in Expression (2) are considered to be given.

The propagation of electromagnetic waves in a medium with zero conductivity and with $\mu = \epsilon = 1$ is described by the Maxwell system of equations

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \operatorname{rot}\mathbf{H}, \qquad \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -\operatorname{rot}\mathbf{E} \tag{3}$$

System (3) with boundary conditions (2) and with given initial conditions allows one to determine the laws of propagation of electromagnetic waves.

In order to find the electric and magnetic fields and the current distribution in the region of motion behind the shock front, conditions (1) should be used, and also the equations of magnetohydrodynamics [4].

Below we will deal with spherical shock waves.

Let the initial magnetic field be represented by $\mathbf{H}_0$, the initial electrical field by $\mathbf{E}_0$. We introduce spherical coordinates r, θ, ϕ, the angle θ being reckoned from the direction of the vector $\mathbf{H}_0$. Bearing in mind that $\mathbf{v}$ and $\mathbf{D}$ are directed along a radius, the boundary conditions (1) and (2) can be resolved as follows:

$$H_{r1} = H_{r2}, \qquad H_{\theta 1} = H_{\theta 2}, \qquad H_{\varphi 1} = H_{\varphi 2} \tag{4}$$

$$E_{\theta 1} = \frac{1}{c}(v_2 - D) H_{\varphi 1}, \qquad E_{\varphi 1} = -\frac{1}{c}(v_2 - D) H_{\theta 1} \tag{5}$$

In view of the fact that for strong shock waves in a perfect gas with ratio of specific heats γ the relation $v_2 = 2D/(\gamma + 1)$ is valid, conditions (4) and (5) can be written thus:

$$E_{\theta 1} = -\frac{\alpha}{c} D H_{\varphi 1}, \qquad E_{\varphi 1} = \frac{\alpha}{c} D H_{\theta 1} \qquad \left(\alpha = \frac{\gamma - 1}{\gamma + 1}\right) \tag{6}$$

We deal with some examples of solutions to the problem of electromagnetic wave propagation.

1. Let $\mathbf{E}_0 = 0$, $\mathbf{H}_0 \neq 0$. We assume that the law of shock-wave motion is given by the expressions

$$D = \frac{dr_2}{dt} = \frac{c\varphi(\xi_2, r_2)}{\alpha\psi(\xi_2, r_2)}, \qquad \xi_2 = ct - r_2, \qquad r_2(0) = 0 \tag{7}$$

$$\varphi(\xi_2, r_2) = \xi_2 \sum_{k=0}^{m} g_k \xi_2^{\,k} [\xi_2 + (k+2) r_2], \qquad 0 \leqslant m < \infty$$

$$\psi(\xi_2, r_2) = r_2^2 - \xi_2 \sum_{k=0}^{m} g_k \xi_2^{\,k} [(k+2) r_2 + \xi_2 + \xi_2^2 r_2^{-1} (k+3)^{-1}]$$

Here g_k are constants.

In this case the solution to the problem is given by the formulas

$$E_r = E_\theta = 0, \qquad H_\varphi = 0$$

$$
\begin{aligned}
E_\varphi &= H_0 \sin\theta \frac{\xi}{r^2} \sum_{k=0}^{m} g_k \xi^k [\xi + (k+2) r] \\
H_r &= -\frac{2H_0}{r^3} \xi^2 \cos\theta \sum_{k=0}^{m} \frac{g_k}{k+3} \xi^k [\xi + (k+3) r] + H_0 \cos\theta \\
H_\theta &= -\frac{H_0}{r} \xi \sin\theta \sum_{k=0}^{m} g_k \xi^k \left[k + 2 + \frac{\xi}{r} + \frac{\xi^2}{(k+3) r^2}\right] - H_0 \sin\theta
\end{aligned}
\tag{8}
$$

Note that the value $\xi = 0$ corresponds to the electromagnetic wave front. From solution (8) it follows that for $\xi = 0$ the components of vectors $\mathbf{E}$ and $\mathbf{H}$ take their initial values.

2. Suppose $D = D_0 = \text{const}$, $\mathbf{H}_0 \neq 0$, $\mathbf{E}_0 \neq 0$, and for simplicity we assume the vectors $\mathbf{H}_0$ and $\mathbf{E}_0$ to be mutually perpendicular.

A strong shock wave at constant velocity can, for instance, be caused by a spherical piston expanding in a gas from some point (taken as the origin) at constant velocity [5].

In this case the problem is a similarity one and its accurate solution may be found by introducing the independent similarity variable $\lambda = r/D_0 t$ by separating variables in system (3) and solving a system of ordinary linear differential equations with subsequent choice of arbitrary constants from the boundary conditions; we only give the final result:

$$
\begin{aligned}
H_r &= H_0 A \left(B + \frac{2}{\delta^2} \frac{1}{\lambda} - \frac{2}{3\delta^4} \frac{1}{\lambda^3}\right) \cos\theta \\
H_\theta &= H_0 A \left(-B - \frac{1}{\delta^2} \frac{1}{\lambda} - \frac{1}{3\delta^4} \frac{1}{\lambda^3}\right) \sin\theta + E_0 A_1 \left(-\frac{1}{\delta} + \frac{1}{\delta^3} \frac{1}{\lambda^2}\right) \cos\varphi \\
H_\varphi &= -E_0 A_1 \left(-\frac{1}{\delta} + \frac{1}{\delta^3} \frac{1}{\lambda^2}\right) \sin\varphi \cos\theta \\
E_r &= E_0 A_1 \left(-B_1 - \frac{2}{\delta^2} \frac{1}{\lambda} + \frac{2}{3\delta^4} \frac{1}{\lambda^3}\right) \sin\varphi \sin\theta \\
E_\theta &= -E_0 A_1 \left(B_1 + \frac{1}{\delta^2} \frac{1}{\lambda} + \frac{1}{3} \frac{1}{\mu^4} \frac{1}{\lambda^3}\right) \sin\varphi \cos\theta \\
E_\varphi &= H_0 A \left(-\frac{1}{\delta} + \frac{1}{\delta^3} \frac{1}{\lambda^2}\right) \sin\theta - E_0 A_1 \left(B_1 + \frac{1}{\delta^2} \frac{1}{\lambda} + \frac{1}{3\delta^4} \frac{1}{\lambda^3}\right) \cos\varphi
\end{aligned}
\tag{9}
$$

where

$$
A = \frac{3\alpha\delta^4}{\alpha(4\delta^3 - 3\delta^2 - 1) - 3(1 - \delta^2)}, \qquad B = \frac{-3(1 - \delta^2) - \alpha(1 + 3\delta^2)}{3\alpha\delta^4} \qquad \left(\delta = \frac{D_0}{c}\right)
$$

$$
A_1 = \frac{3\delta^4}{1 + 3(\alpha + 1)\delta^2 - 4\delta^3 - 3\alpha\delta^4}, \qquad B_1 = \frac{-1 - 3(\alpha + 1)\delta^2 + 3\alpha\delta^4}{3\delta^4}
$$

The validity of Equation (9) can be confirmed by direct check.

It should be noted that solution (9) for the case $E_0 = 0$ could be obtained from solution (8) by putting $m = 0$.

3. A solution to the Maxwell equations written down in Formulas (8) can be used for finding, approximately, the electromagnetic wave parameters for laws of $D(t)$ other than (7).

To do this we must substitute in (8) constants g_k found from the conditions of approximation of the relations $D(t)$ or $D(r_2)$, using Expression (7). In particular, this expression can be used when $D(t)$ and $r_2(t)$ are given in tabular form. It should be noted in this connection that if the initial position of the electromagnetic wave is represented by $r = r_0$, the variable ξ in solution (8) should take the form

$$\xi = r - r_0 - ct$$

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